1 Appendix: Inventory Dissipation Formalism for BESS under ECME

This appendix formalizes an ECME-compatible coupling between a market (contractual) state and a physical (battery) state for a Battery Energy Storage System (BESS). The core mechanism is a *state split* together with a *dissipative reaction operator* that relaxes contractual imbalances subject to physical rate limits, and a *thermodynamic synchrony* coefficient that modulates operational behavior in a smooth, consensus-friendly way.

1.1 State Split: Contractual vs. Physical Endowments

Consider an agent i and a distinguished physical asset e (energy). We define two endowment states:

- **Contractual endowment** $x^{\mathrm{ctr}}_{i,e}(t)$: the legally binding inventory recorded and updated by the market (e.g., blockchain state transitions).
- **Physical endowment** $x^{\mathrm{phys}}_{i,e}(t)$: the physically realizable inventory in the battery (e.g., electrochemical state-of-charge mapped into units of e).

The split enables the market to represent positions and trades independently from the battery's finite actuation speed. The coupling is expressed as a *reaction* dynamics in discrete time (or epochs), while the ECME exchange engine operates on the contractual state.

1.2 Reaction Dynamics: Rate-Limited Physical Relaxation

Let $\Delta t > 0$ be a reaction time step and let $\kappa_{i,e} \geq 0$ be the maximum physical charge/discharge rate (C-rate converted into units of e per unit time). Define the discrepancy

$$\Delta_{i,e}(t) \;:=\; \left| x^{\mathrm{ctr}}_{i,e}(t) - x^{\mathrm{phys}}_{i,e}(t) \right|. \tag{1}$$

We postulate a dissipative, rate-limited pursuit dynamics:

$$x^{\mathrm{phys}}_{i,e}(t+\Delta t) = x^{\mathrm{phys}}_{i,e}(t) + \mathrm{clip}\Big(x^{\mathrm{ctr}}_{i,e}(t) - x^{\mathrm{phys}}_{i,e}(t),\; -\kappa_{i,e}\Delta t,\; +\kappa_{i,e}\Delta t \Big), \tag{2}$$

where $\mathrm{clip}(z, a, b) = \min(\max(z, a), b)$. If the battery has capacity $C_{i,e} > 0$, the physical state is additionally projected into feasibility:

$$x^{\mathrm{phys}}_{i,e}(t+\Delta t) \leftarrow \mathrm{clip}\big(x^{\mathrm{phys}}_{i,e}(t+\Delta t),\, 0,\, C_{i,e} \big). \tag{3}$$

Interpretation. Equation (2) is a *relaxation operator* that reduces the contractual–physical mismatch subject to the finite response speed of the BESS. In reaction–diffusion terms, trading perturbs the contractual state (diffusion), while the battery physics dissipates mismatch energy (reaction).

1.3 Thermodynamic Synchrony Coefficient

We define a scalar *thermodynamic synchrony* (availability) coefficient $\phi_{i,e}(t) \in (0, 1]$ as a monotone function of mismatch, chosen to avoid transcendental functions:

$$\phi_{i,e}(t) \;:=\; \frac{1}{1 + \lambda_{i,e}\, \Delta_{i,e}(t)}, \qquad \lambda_{i,e} > 0. \tag{4}$$

Properties.

- If $\Delta_{i,e}(t) = 0$ then $\phi_{i,e}(t) = 1$ (perfect synchrony).
- As $\Delta_{i,e}(t) \to \infty$, $\phi_{i,e}(t) \to 0$ (high stress / low availability).
- The functional form is rational and thus implementable in fixed-point arithmetic with deterministic rounding.

1.4 Preference Modulation via Synchrony

In the ECME framework, exchange decisions are parameterized by Cobb–Douglas weights (pairwise or derived from a multivariate exponent vector). Let $\alpha_i^{(a,b)} \in (0,1)$ denote agent i's *pairwise* preference weight on asset b in the two-good restriction (a,b). Introduce a conservative (risk-averse) anchor $\alpha_{\mathrm{safe},i}^{(a,b)} \in (0,1)$ representing the strategy under severe physical stress.

We define an effective preference used for operational execution:

$$\alpha_{\mathrm{eff},i}^{(a,b)}(t) = \phi_{i,e}(t)\,\alpha_i^{(a,b)} + \big(1-\phi_{i,e}(t)\big)\,\alpha_{\mathrm{safe},i}^{(a,b)}. \tag{5}$$

Interpretation. When the battery is synchronized ($\phi \approx 1$), the market sees the agent's intended preference. When mismatch grows ($\phi \downarrow$), the agent smoothly transitions toward a conservative mode (e.g., reduced willingness to sell energy or increased retention).

1.5 Temporal Solvency: Energy Debt Constraint

Define the *energy debt* as the signed discrepancy

$$\delta_{i,e}(t) \;:=\; x_{i,e}^{\mathrm{ctr}}(t) - x_{i,e}^{\mathrm{phys}}(t). \tag{6}$$

To prevent the ECME market from creating physically infeasible obligations, impose a debt limit

$$\delta_{i,e}(t) \le L_{i,e}, \qquad L_{i,e} \ge 0, \tag{7}$$

where $L_{i,e}$ is a policy parameter (possibly zero in a strictly conservative regime).

Regimes.

- **Risk-averse:** $L_{i,e} = 0$ enforces $x_{i,e}^{\mathrm{ctr}}(t) \le x_{i,e}^{\mathrm{phys}}(t)$, i.e., no selling of unreacted (uncharged) energy.
- **Bounded forward exposure:** $L_{i,e} > 0$ allows limited forward commitments, interpretable as collateralized or margin-constrained obligations.

1.6 Mismatch Free Energy and Dissipation

To make the thermodynamic interpretation explicit, define a mismatch potential (free energy) for agent i:

$$\mathcal{F}_{i,e}(t) := \frac{1}{2}\,\Delta_{i,e}(t)^2 = \frac{1}{2}\big(x_{i,e}^{\mathrm{ctr}}(t) - x_{i,e}^{\mathrm{phys}}(t)\big)^2. \tag{8}$$

Reaction–diffusion reading. Trading activity in ECME updates x^{ctr} and may inject energy into the mismatch potential $\mathcal{F}$, while the physical reaction operator (2) dissipates $\mathcal{F}$ over time, constrained by $\kappa_{i,e}$. This yields an endogenous liquidity effect: physical inertia induces a smooth reduction in market aggressiveness (via (5)) or market eligibility (via (7)) without requiring discrete kill switches.

1.7 Implementation Alignment with ECME

The above formalism is compatible with ECME under either of the following coupling choices:

1. **Preference coupling:** use $\alpha_{\mathrm{eff},i}^{(a,b)}(t)$ in the pairwise execution rule (endogenous price/MRS computation) while keeping the contractual state as the settlement source of truth.
2. **Eligibility coupling:** enforce the debt constraint (7) (and optionally a threshold on $\phi_{i,e}$) as a prerequisite for insertion into, or execution from, ECME's heaps.

In both cases, the *state split* ensures that the diffusion (free trade) phase remains economically expressive, while the reaction (physical) phase enforces conservation and feasibility through a dissipative invariant mechanism.